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Advanced Algebraic Topology

Midterm Exam, May 9

1. (6 points) Let **Ab** the the category of abelian groups and group homomorphisms. Let $\mathrm{id}\colon \mathbf{Ab} \to \mathbf{Ab}$ be the identity functor. Let $F\colon \mathbf{Ab} \to \mathbf{Ab}$ be the functor, where
$$F(G) = \mathrm{Hom}(\mathbb{Z}, G),$$
and for a group homomorphism $f\colon G \to H$,
$$F(f)\colon \mathrm{Hom}(\mathbb{Z}, G) \to \mathrm{Hom}(\mathbb{Z}, H), \;\; k \mapsto f \circ k.$$
Show that there is a natural equivalence $\mathrm{id} \to F$.

2. (6 points) Let X and Y be topological spaces. Let the integral homology groups of X be
$$H_0(X) = \mathbb{Z} \oplus \mathbb{Z}, \;\; H_1(X) = \mathbb{Q}, \;\; H_2(X) = \mathbb{Z}/2\mathbb{Z}, \;\; H_p(X) = 0, \;\; \text{for } p > 2$$
and let the integral homology groups of Y be
$$H_0(Y) = \mathbb{Z}, \;\; H_1(Y) = \mathbb{Z}/3\mathbb{Z}, \;\; H_2(Y) = \mathbb{Z}/4\mathbb{Z}, \;\; H_p(Y) = 0,$$
for $p > 2$. Calculate the integral homology groups of $X \times Y$.

3. (6 points) Calculate the cohomology groups $H^p(\mathbb{R}P^3 \vee \mathbb{R}P^2; \mathbb{Z}/4\mathbb{Z})$, $p \geq 0$.

4. (6 points) Show that $\mathbb{S}^2 \vee \mathbb{S}^3$ is not a retract of $\mathbb{S}^2 \times \mathbb{S}^3$.

$(1 + x^2)(1 + x^3)$

$= 1 + x^3 + x^2 + x^5$