de Rham theory Spring 2023
Exam March 1, 2023
Exam time 10.15-12.00

1.(6p.) Let $\varphi\colon V \to W$ be a surjective linear map between finite dimensional vector spaces of dimension at least 2. Show that $\varphi^*\colon \mathrm{Alt}^2(W) \to \mathrm{Alt}^2(V)$ is injective.

2.(6p.) Let n and m be integers satisfying $m > n$. Let also $U \subset \mathbb{R}^n$ be a non-empty open set and $\iota\colon U \to \mathbb{R}^m$ the inclusion map $(x_1, \dots, x_n) \mapsto (x_1, \dots, x_n, 0, \dots, 0)$. Let also $y = (y_1, \dots, y_m)\colon \mathbb{R}^m \to \mathbb{R}^m$ be the identity map and let $1 \leq j_1 < \cdots < j_k \leq m$ be integers. Show that $\iota^*(dy_{j_1} \wedge \cdots \wedge dy_{j_k}) = 0$ if $j_k > n$.

3.(6p.) Let $E = [-1, 1] \times \{0\} \times \{0\} \subset \mathbb{R}^3$. Calculate $\dim H^2(\mathbb{R}^3 \setminus E)$.