

**de Rham theory Spring 2023**  
**Exam May 3, 2023**  
**Exam time 10.15-12.00**

- 1.(6p.) Let  $M$  and  $N$  be smooth manifolds and let  $\pi_M: M \times N \rightarrow M$  and  $\pi_N: M \times N \rightarrow N$  be projections. Let  $p \in M$  and  $q \in N$ . Show that the mapping  $f: T_{(p,q)}(M \times N) \rightarrow (T_p M) \times (T_q N)$ ,  $v \mapsto ((D\pi_M)_{(p,q)}(v), (D\pi_N)_{p,q}(v))$ , is a linear isomorphism.
- 2.(6p.) Let  $k > 0$ . Calculate  $H^\ell(\mathbb{S}^k \times \mathbb{S}^1)$  for  $\ell \in \{0, \dots, k+1\}$ .
- 3.(6p.) Let  $0 < k < n$ . Calculate  $H_c^\ell(\mathbb{R}^k \times \mathbb{S}^{n-k})$  for  $\ell \in \{0, \dots, n\}$ .