

de Rham theory Spring 2023

Exam March 2, 2023

Exam time 14.15-16.00

1.(6p.) Let $U \subset \mathbb{R}^2$ be a non-empty open set and $f: U \rightarrow \mathbb{R}^4$ be the map $(p_1, p_2) \mapsto (4p_1, p_2, 2p_2, 2p_1)$. Let also $y = (y_1, y_2, y_3, y_4): \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be the identity map. Show that $f^*(dy_1 \wedge dy_2 + dy_3 \wedge dy_4) = 0$.

2.(6p.) Find functions $a, b \in C^\infty(\mathbb{R}^2)$ for which the 1-form $\tau = adx_1 + bdx_2$ satisfies $d\tau = dx_1 \wedge dx_2$.

3.(6p.) Calculate $\dim H^2(\mathbb{R}^3 \setminus \{(1, 0, 0), (-1, 0, 0)\})$.