Department of Mathematics and Statistics
Dependence Logic
Course exam 14.12.2023

1. Show that there exists a dependence logic sentence ϕ with vocabulary $\tau = \emptyset$ such that for all finite structures $\mathcal{M}$:

$$\mathcal{M} \models \phi \Leftrightarrow |\mathrm{Dom}(\mathcal{M})| \text{ is odd.}$$

If you wish, you may use the fact that $\mathrm{FO}(=(...)) \equiv_s \Sigma_1^1$.

2. Let $\phi \in \mathrm{FO}(\subseteq)$. Show that the following logical equivalence holds:

$$\phi \equiv \phi \vee \phi. \tag{1}$$

Construct a formula $\psi \in \mathrm{FO}(=(...))$ for which the equivalence of (1) fails.

3. Give the truth condition for independence atoms $\vec{x} \perp_{\vec{z}} \vec{y}$, that is, when does $\mathcal{M} \models_X \vec{x} \perp_{\vec{z}} \vec{y}$ hold? Let x, y, z be pairwise distinct variables. Construct formulas ϕ, ψ and θ of independence logic such that

$$(\phi \wedge \psi) \vee \theta \not\equiv (\phi \vee \theta) \wedge (\psi \vee \theta).$$

4. Show for all structures $\mathcal{M}$ and teams X, if $\mathcal{M} \models_X \vec{x} \perp \vec{y}$ and $\mathcal{M} \models_X \vec{x}\vec{y} \perp \vec{z}$, then $\mathcal{M} \models_X \vec{x} \perp \vec{y}\vec{z}$. Recall that $\vec{x} \perp \vec{y}$ is an abbreviation for the independence atom of the form $\vec{x} \perp_{\vec{z}} \vec{y}$, where $\vec{z} = \emptyset$.