University of Helsinki
Bachelor's programme in mathematical sciences
MAT21031 Elements of set theory II
Course exam
7.5.2024

No calculators, charts or other extra material allowed.

Leave a few empty rows for grading notes at the top of the first page of your answer sheet.

1. Show directly from the definitions of *cardinal arithmetic* that

$$(\kappa \cdot \lambda)^\mu = \kappa^\mu \cdot \lambda^\mu.$$

2. Prove that the following forms of the Axiom of Choice are equivalent:
   (i) For any sets C and D, $C \preccurlyeq D$ or $D \preccurlyeq C$ (or both).
   (ii) For any set A there is a well-ordering $<$ of A.

3. In *ordinal arithmetic*, simplify the following:
   (a) $\omega + 2 \cdot \omega^3$,
   (b) $\omega^2 \cdot \omega^\alpha$,
   (c) $\bigcup\{\alpha^\delta : \delta < \omega_1\} + \omega_1$.

4. Show that for any sets a, b,
   (a) $\operatorname{rank}\{a, b\} = \max(\operatorname{rank} a, \operatorname{rank} b)^+$,
   (b) $\operatorname{rank} \bigcup a \subseteq \operatorname{rank} a$.