University of Helsinki
Bachelor's programme in mathematical sciences
MAT21030 Elements of set theory I
Course exam
4.3.2024

No calculators, charts or other extra material allowed.

Leave a few empty rows for grading notes at the top of the first page of your answer sheet.

1. (a) Simplify $(\bigcup A) \cap (\bigcup B)$, where $A = \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$ and $B = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}, \{\{\emptyset\}\}\}\}$.
   (b) Prove or disprove (i.e., provide a proof or a counter example): For any sets A and B
   $$\mathcal{P}A \cup \mathcal{P}B = \mathcal{P}(A \cup B)$$
2. Show that every natural number is a transitive set.
3. Show that addition of reals is commutative, i.e,
   $$x +_{\mathbb{R}} y = y +_{\mathbb{R}} x$$
   for all $x, y \in \mathbb{R}$. State without proof the results from earlier levels of the construction that you use in your proof.
4. Show that for any $n \in \omega$, any proper subset of n is equinumerous to some m less than n. (Hint: induction)