

Final exam Elements of Set Theory Spring 2023

April 27, 2023

1. Describe the construction of the real numbers starting with the natural numbers. Give as much detail as possible. Prove that the addition of the reals is commutative. Show that the operation of multiplication on the rationals is well-defined.
2. Prove the recursion theorem on ω . Recall the set \mathcal{H} , defined as the set of all acceptable functions. Prove that \mathcal{H} is a set.
3. Prove that $\kappa \cdot \kappa = \kappa$ for κ an infinite cardinal.
4. Compute the rank of the set of all functions from the rational numbers to the rational numbers.