

University of Helsinki
Bachelor's programme in mathematical sciences
MAT21031 Elements of set theory II
Retake exam
22.5.2024

No calculators, charts or other extra material allowed.

*If you are retaking just the course exam (and want your grade to take exercise points into account), clearly mark this on your answer sheet and answer **FOUR** questions. You can choose which one you omit.*

*If you are retaking the whole course (without exercise points), answer all **FIVE** questions.*

If it is unclear from the answer sheet which one of the above alternatives is chosen, the exam is graded as a five question general exam.

Leave a few empty rows for grading notes at the top of the first page of your answer sheet.

1. Let κ be an infinite cardinal. Show that (in cardinal arithmetic)

$$\aleph_0^{(\kappa + \aleph_0)} \cdot \kappa = 2^\kappa.$$

2. Prove the well-ordering principle from Zorn's lemma (without using other forms of AC).
3. Let A be a set of cardinal numbers. Show that $\alpha = \bigcup A$ is also a cardinal number. □
4. (a) How is exponentiation of ordinals defined recursively?
(b) Show by induction that for ordinals α, β, γ ,

$$\alpha^\beta \cdot \alpha^\gamma = \alpha^{\beta + \gamma}.$$

5. Compute the rank of the set ${}^4\omega$ of functions from 4 to ω .