University of Helsinki
Bachelor's programme in mathematical sciences
MAT21030 Elements of set theory I
General exam
8.11.2023

No calculators, charts or other extra material allowed.

1. Simplify
   (a) $\bigcup\{\{3,4\},\{\{3\}.\{4\}\},\{3,\{4\}\},\{\{3\},4\}\}$,
   (b) $\bigcap\{\mathcal{P}\mathcal{P}\mathcal{P}\{\emptyset\},\mathcal{P}\{\emptyset\},\mathcal{P}\mathcal{P}\emptyset\}$.
2. Show that if $A \subseteq B$, then $\bigcup A \subseteq \bigcup B$.
3. Define the rank of a set c to be the least α such that $c \subseteq V_\alpha$ (constructed without atoms).
   (a) Compute the rank of $\{\{\emptyset\}\}$.
   (b) What is the rank of 4 (as defined on the course)?
4. Show that if a set A satisfies $A \subseteq \mathcal{P}A$, then A is transitive.
5. Give a short (less than one page in normal handwriting) description of the steps of the set theoretic construction of the reals. Concentrate on the definitions of the elements (you can omit the arithmetic part this time).