Exam: Set Theory, Spring 2019

May 13, 2019

- Prove that if there is a $1-1$ mapping f of a set A into a set B, and there is a $1-1$ mapping g of B into A, then A and B are equinumerous.
- State and prove the equivalence of two statements of the axiom of choice.
- Tell the story of the construction of the reals, starting with the definition of the natural numbers and passing through the successive definitions of the integers, the rational numbers and the reals. How are addition and multiplication defined in all of these structures? Prove that for Dedekind cuts, i.e. two real numbers r and s, $r+s=s+r$. Prove that the embedding of the rationals into the reals is injective.
- State and prove the transfinite recursion theorem, given a well-ordering $< A, <_A >$, and a functional relation $\gamma(x,y)$. Use this theorem to define the $\aleph_\alpha$'s. State the generalisation of this theorem with the class of ordinals "replacing" the well-ordering $< A, <_A >$.
- Calculate $\aleph_0 \cdot 2^{\aleph_0}$