University of Helsinki
Department of Mathematics and Statistics
Finite model theory
Course exam 12.12.2024

1. Let τ be a finite and relational vocabulary and $k \geq 1$. Suppose $\mathfrak{A}$ and $\mathfrak{B}$ are finite τ-models.

a) Give the definition of $\mathfrak{A}$ and $\mathfrak{B}$ being partially isomorphic up to k variables ($\mathfrak{A} \cong^k \mathfrak{B}$).

b) Show that connectivity of finite graphs cannot be defined in $\mathcal{L}^2_{\infty,\omega}$.

2. Let τ be as above. Show that there is no τ-sentence φ of FO such that for all finite τ-models $\mathfrak{A}$ holds:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is odd.}$$

3. Let $\Sigma = \{a, b, c\}$ and define

1. $L_1 = \{w \in \Sigma^+ \mid |w| \text{ is even}\}$
2. $L_2 = \{w \in \Sigma^+ \mid w = a^k bc^l \text{ for some } k, l \geq 1\}$

Above a^k denotes a string $aa \cdots a$ of length k. Show that both languages L_1 and L_2 can be defined in MSO. *You may use Büchi's theorem in your solution.*

4. When is a sentence of the form $[\mathrm{TC}_{x,y}\varphi]cc'$ true in a model $\mathfrak{A}$, where c and c' are constant symbols? Construct a sentence φ of FO(TC) such that for all finite *ordered* models $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is odd.}$$