

University of Helsinki  
Department of Mathematics and Statistics  
Finite model theory  
Final exam 15.12.2022

1. Let  $\tau$  be a finite and relational vocabulary and  $k \in \mathbb{N}$ . Suppose  $\mathfrak{A}$  and  $\mathfrak{B}$  are finite  $\tau$ -models.
  - a) Give the definition of  $\mathfrak{A}$  and  $\mathfrak{B}$  being partially isomorphic up to  $k$  variables ( $\mathfrak{A} \cong^k \mathfrak{B}$ ).
  - b) Show that the connectivity of finite graphs cannot be defined in  $\mathcal{L}_{\infty, \omega}^2$ .
2. Let  $\tau = \emptyset$ . Show that there is no  $\tau$ -sentence  $\varphi$  of FO such that for all finite  $\tau$ -models  $\mathfrak{A}$  holds:

$$\mathfrak{A} \models \varphi \Leftrightarrow |\text{Dom}(\mathfrak{A})| \text{ is even.}$$

3. Let  $\Sigma = \{a, b, c\}$  and define

1.  $L_1 = \{w \in \Sigma^+ \mid |w| = 0 \pmod{3}\}$

2.  $L_2 = \{w \in \Sigma^+ \mid w = a^k b c^l \text{ for some } k, l \geq 1\}$

- a) Give the definition of a word model for  $w \in \Sigma^+$  words.
- b) Show that both languages  $L_1$  and  $L_2$  can be defined in MSO.

Note: *You may assume Büchi's theorem to be known in your solution.*

4. When is a sentence of the form  $[\text{TC}_{x,y}\varphi]cc'$  true in a model  $\mathfrak{A}$ , where  $c$  and  $c'$  are constant symbols? Construct a sentence  $\varphi$  of FO(TC) such that for all finite *ordered* models  $\mathfrak{A}$ :

$$\mathfrak{A} \models \varphi \Leftrightarrow |\text{Dom}(\mathfrak{A})| \text{ is even.}$$