

Department of Mathematics and Statistics
Homotopy theory
1st midterm exam 26.10.2023

1. Explain briefly the definition of the fundamental group: what are the elements of the group, how are they constructed? How is the group operation defined? Present also the definition of the induced homomorphism and its' most important properties. Here you don't have to give any proofs; if needed, you can for example use the phrase "it can be proved that ...".
2. Let X and Y be topological spaces. Which of the following claims are true? Give a proof or a counterexample.
 - a) If X is contractible, then every continuous function $f: X \rightarrow Y$ is null homotopic.
 - b) If X is contractible, then any two continuous functions $f, g: X \rightarrow Y$ are homotopic with each other.
 - c) If Y is contractible, then every continuous function $f: X \rightarrow Y$ is null homotopic.
 - d) If Y is contractible, then any two continuous functions $f, g: X \rightarrow Y$ are homotopic with each other.
3.
 - a) Calculate $\pi(X)$, where $X = \mathbb{R}^3 \setminus \{(0, 0, z) \in \mathbb{R}^3 \mid z \in \mathbb{R}\}$.
 - b) Suppose that $f: S^1 \rightarrow S^1$ is a continuous function, which is null homotopic. Prove that f has a fixed point.
4. Formulate and prove the Seifert-van Kampen Theorem (not the general version, but the special case concerning simply connectedness which was proved in the lectures).