University of Helsinki
Department of Mathematics and Statistics

Kankaanrinta

Homotopy theory, MAST 31020
Midterm Exam 2, May 7, 2019

1. (6 points)

(1) Use homotopy groups to show that there is no retraction $\mathbb{R}P^n \to \mathbb{R}P^k$ if $n > k > 0$.

(2) Show that $\pi_7(\mathbb{S}^4)$ contains an element of infinite order.

2. (6 points)

(1) Let $p\colon X \to Y$ be a fibration and suppose that $s\colon X \to W$ and $g\colon W \to Y$ are maps with $g \circ s = p$. Assume g is injective. Show that s is a fibration.

(2) Let A be a subspace of a space X. Let $i\colon A \hookrightarrow X$ be the inclusion. Assume $i_*\colon \pi_1(A) \to \pi_1(X)$ is injective. Show that $\pi_2(X, A)$ is abelian.

3. (6 points) Let A be a closed, contractible subspace of a topological space X. Assume the inclusion $j\colon A \hookrightarrow X$ is a cofibration. Show that the quotient map $q\colon X \to X/A$ is a homotopy equivalence.

4. (6 points)

(1) State the Blakers - Massey theorem (i.e., the homotopy excision theorem).

(2) Define the infinite symmetric product $SP(X)$ of a pointed space X.