MAST31215 Introduction to Continuous Logic
Master's programme in Mathematics and Statistics
University of Helsinki
General examination 8.1.2025

Only answer four questions

1. Prove that for each L-term $t(x_1, \ldots, x_n)$ and each L-formula $\varphi(x_1, \ldots, x_n)$ there are functions Δ_t and Δ_φ from $(0, 1]$ to $(0, 1]$ such that for any L-structure $\mathcal{M}$, Δ_t is a modulus of uniform continuity for the function $t^{\mathcal{M}} : M^n \to M$ and Δ_φ is a modulus of uniform continuity for the predicate $\varphi^{\mathcal{M}} : M^n \to [0, 1]$.

2. Show that for any vocabulary L of metric structures there is a dense (in the logical distance metric) set of L-formulas in *prenex normal form*, i.e. in the form
$$Q^1_{x_1} Q^2_{x_2} \ldots Q^n_{x_n} \psi$$
where ψ is quantifier free and each Q^i is either sup or inf.

3. Show that compactness (of the structure) is not axiomatizable in continuous logic.

4. State and prove the downward Löwenheim-Skolem theorem for continuous logic.

5. Consider the structure $\mathcal{M} = ([0, 1], f)$ where the metric on $[0, 1]$ is the ordinary metric on the reals, and f is some continuous function $[0, 1] \to [0, 1]$. Show that the range of f is a definable set in $\mathcal{M}$.