University of Helsinki
Introduction to Logic II
Course Examination, May 9, 2022, 2,5 hours
Notes, tables of formulae, and calculators are not allowed in the exam.

1. Using Tarski's truth definition show that the following formula is valid

$$\forall x_0 R_0(x_0, c_0) \to \forall x_0 \exists x_1 R_0(x_0, x_1)$$

2. Let $\mathcal{M} = (\mathbb{N}, R_0^{\mathcal{M}})$, where $R_0^{\mathcal{M}} = \{(a, b) \in \mathbb{N}^2 \mid a < b\}$.
   Show that the set $\{0\}$ can be defined by a formula in $\mathcal{M}$.

3. Prove by natural deduction $\exists x_0 A \to \exists x_0 B$
   from the formula $\forall x_0 (A \to B)$.
   Explicitly check in writing that used quantifier rules apply.

4. Consider the structures $\mathcal{M}$ and $\mathcal{M}'$ for the vocabulary $\{R_0\}$. The universes of both structures $\mathcal{M}$ and $\mathcal{M}'$ consist of the numbers $\{0, 1, 2, 3\}$ and the interpretations of the symbols are

$$R_0^{\mathcal{M}} = \{(0, 1), (0, 2), (1, 3), (2, 3)\}$$

   and

$$R_0^{\mathcal{M}'} = \{(0, 1), (0, 2), (3, 1), (3, 2)\}.$$

   Are the structures $\mathcal{M}$ and $\mathcal{M}'$ isomorphic?
   Complete solution requires explicitly checking the conditions for being isomorphic.