Measure and Integral

Department of Mathematics and Statistics

Course Exam, 19.10.2026

Extra materials are not allowed in the exam

1. (a) Give the definition of Lebesgue–measurable subset of $\mathbb{R}^n$.

   (b) Give the definition of a measurable mapping.

2. Show that $E \subset \mathbb{R}^n$ is Lebesgue meaasurable in $\mathbb{R}^n$ if and only if for each $\varepsilon > 0$ there exsists measurable A and B such that $A \subset E \subset B$ and $m(B \setminus A) < \varepsilon$.

3. (From Homework). Are the following claims true? If yes, give a proof, and if you think they are false, please, give a counterexample.

   (a) If $m^*(A) > 0$ then A contains an open set which is non–empty.

   (b) If $m^*(A) < \infty$ then A is bounded, i.e there exsist $R > 0$ such that $A \subset B(0, R) = \{x \in \mathbb{R}^n;\ |x| < R\}$.

4. Let $A_n = \{(x, y) \in \mathbb{R}^2;\ |x| < 1/n,\ y \in \mathbb{R}\}$ for all $n \in \mathbb{N}$. Assume that $f : \mathbb{R}^2 \to \mathbb{R}$ is integrable over $\mathbb{R}^2$. Prove that

$$\lim_{n\to\infty} \int_{A_n} f = 0.$$

5. Assume that $f : \mathbb{R} \to [0, \infty]$ is a measurable function for which

$$\int_{\mathbb{R}} f < \infty.$$

   Let $E(x) = (-\infty, x] \subset \mathbb{R}$. Prove that the function

$$\mathbb{R} \ni x \mapsto \int_{E(x)} f$$

   is continuous.