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Dept of Mathematics and Statistics
MAT22001 Probability IIa
Moodle home exam
23.10.2020 at 12:00-15:00

Return your solutions to the Moodle submission folder by 15:00.

1. Let the random variable X be continuously distributed with the probability density function f_X. Derive the probability density function f_Y of the random variable $Y = |X - 1|$. If $X \sim U(-1, 0)$, what is the distribution of Y?

2. Let X_1 and X_2 be independent random variables with
$$EX_1 = 2, \quad EX_2 = -1, \quad \operatorname{var} X_1 = 1 \quad \text{and} \quad \operatorname{var} X_2 = 2.$$
Define
$$Y = 2 - X_1 + 2X_2 \quad \text{and} \quad Z = -3 + 2X_1 - X_2.$$
Calculate EY, EZ, $\operatorname{var} Y$, $\operatorname{var} Z$ and $\operatorname{cov}(Y, Z)$.

3. Let $X \sim U(-1, 1)$ and Y be a random variable with $P(Y = -1) = P(Y = 1) = \frac{1}{2}$. Prove that if $X \perp\!\!\!\perp Y$, then $X + Y \sim U(-2, 2)$. *Hint:* Moment generating functions. If you have difficulties to begin with your solution, search the formula for the moment generating function of the uniform distribution from the web, and then derive the formula in the case $X \sim U(-1, 1)$.

4. Prove that the events A and B are independent if and only if $\operatorname{cov}(\mathbf{1}_A, \mathbf{1}_B) = 0$.