

Topology II  
Exam Tuesday October 24, 2023  
Exam time 14.00-17.00

Problems

- p1. Let  $X = \{0\} \cup \mathbb{N}$  and let  $\mathcal{T} = \{\emptyset\} \cup \{\{0\} \cup A : A \subset \mathbb{N}\}$ .
- (a) Show that  $\mathcal{T}$  is a topology in  $X$ .
  - (b) Show that  $\{0\}$  is dense in  $(X, \mathcal{T})$ .
- p2. Let  $(X, \mathcal{T}_X)$  and  $(Y, \mathcal{T}_Y)$  be topological spaces and let  $f: X \rightarrow Y$  be an embedding. Show that the topology  $\mathcal{T}$  induced by  $f$  from  $\mathcal{T}_Y$  is  $\mathcal{T}_X$ .
- p3. Let  $\sim$  be the equivalence relation in  $\mathbb{R}$  having equivalence classes  $\mathbb{Q}$  and  $\mathbb{R} \setminus \mathbb{Q}$ , that is,  $x \sim y$  if either  $\{x, y\} \subset \mathbb{Q}$  or  $\{x, y\} \subset \mathbb{R} \setminus \mathbb{Q}$ . Show that every continuous map  $f: \mathbb{R}/\sim \rightarrow \mathbb{R}$  is constant, where  $\mathbb{R}/\sim$  has the quotient topology.
- p4. Let  $(X, \mathcal{T}_X)$  be a topological space, where  $X = \{0, 1\}$  and  $\mathcal{T}_X = \{\emptyset, \{0\}, X\}$ . For which  $j = 1, \dots, 4$  the product space  $X \times X$  is a  $\mathcal{T}_j$ -space? Justify your answer.