Topology II
Exam 2
Exam December 10, 2024
Exam time 14.00-17.00

Problems

p1. Let X be a separable space, Y a topological space, and let $f\colon X \to Y$ be continuous surjective map. Show that Y is separable.

p2. Let X and Y be topological spaces, $a \in X$, $b \in Y$. Show that

$$Q(a, X) \times Q(b, Y) \subset Q((a, b), X \times Y),$$

where $Q(z, Z)$ is the quasicomponent of $z \in Z$ of a topological space Z.

p3. Suppose $(X_i)_{i \in I}$ is a family of non-empty Hausdorff spaces having the property that $X = \prod_{i \in I} X_i$ is locally compact. Show that there exists a finite set $F \subset I$ having the following property: for $i \in F$, the space X_i is locally compact and, for $i \in I \setminus F$, the space X_i is compact.

p4. Let $(X, \mathcal{T})$ be a regular topological space. Does there exist a maximal topology $\mathcal{T}'$ containg $\mathcal{T}$ for which the space $(X, \mathcal{T}')$ is regular. Here maximality is understood in the sense of inclusion.